ELSEVIER

Journal of Pure and Applied Algebra 144 (1999) 29–65

JOURNAL OF PURE AND APPLIED ALGEBRA

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Smoothness, flatness and deformations of subalgebras ☆

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Communicated by E.M. Friedlander; received 5 February 1997

Abstract

The smoothness of a morphism between noetherian schemes can be recognized in terms of the induced mappings between tangent and obstruction spaces. This observation can be effectively applied in the study of schemes parametrizing certain objects of interest in deformation theory. Strong versions of the classical results on rigidity and stability of subalgebras in finite dimensional Lie algebras are derived as an application. Some special cohomological conditions ensuring rigidity or stability are obtained in case of a field of positive characteristic.

MSC: 14D15; 17B55; 14B10

0. Introduction

Let k be an algebraically closed field, L a finite dimensional Lie algebra over k and M its subalgebra. One can ask several questions concerning the behavior of M with respect to deformations. If every small deformation of M within L is isomorphic to M and the isomorphism can be taken close to the inclusion mapping of M into L then M will be called *rigid* in L. If, moreover, the isomorphism can always be obtained as a conjugation by an element of an algebraic automorphism group $G \subset \operatorname{Aut} L$ close to the identity element then M is said to be *rigid with respect to* G. Suppose now that we vary the multiplication in the ambient Lie algebra L. If every small deformation of L contains a subalgebra obtained as a small deformation of M then M is called *weakly*

☆ The paper was written during the time when the author visited the Universities of Hamburg and Manchester under the sponsorships of Die Deutsche Forschungsgemeinschaft and the Royal Society of Great Britain. The author gratefully acknowledges their support.

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0022-4049/99/$ - see front matter
PII: S0022-4049(99)00072-9